Does the Look Ahead pairing method maximize players receiving the correct color? (Sort of.)

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1 Introduction

Chess tournament directors use pairing algorithms to determine which players face each other in each round. While ultimately pairings depend on a number of factors (which player is in which score group, who is due which color, maximum rating differences, etc.), it is helpful to tournament directors to know what properties these pairing algorithms can provide.

One of the pairing algorithms described in the US Chess Federation's Official Rules of Chess, 7th Edition, is called the Look Ahead method (29E6a). The rule book describes a particular scenario, where it states that if more than half the group is due for the same color, then by avoiding pairings in which *neither* player is due for that color, you *maximize* the number of players receiving their due color.

The particular details of how this algorithm works are not relevant here, but a few background terms are necessary.

- White and black are the colors of pieces which a player may have in a given chess game.
- A due color is either white, black, or neither, and this is based on the number of white/black this player has received as well as their prior round color.
- A score group is all the players who have the same score, who are paired together. Score groups may drop or add a player who has a close but non-identical score, for pairing purposes.

Ultimately, we find that this proposition as they stated it is not true and has a simple counterexample. When an additional restriction is added (the score group contains an even number of players) it becomes true again. I suspect that this was overlooked because only the lowest score group will be an odd size for pairing purposes.

2 Proposition

As above, we know we have players we wish to pair, and we want to give people their due color when possible. The proposition as posed in the rule book is:

Proposition 1. *Avoiding pairings in which neither player is due for that color will maximize the number of players who receive their due color.*

The particular details of the pairing method are not relevant for *how* we achieve this. The objective here is to show that, *if* we do, do we achieve the maximum?

We'll see a counter-example for odd-size groups, then we'll see a proof that we do acheive the maximum for even-size groups.

3 Odd-cardinality score group counter-example

Assume we have a 5-player score group with A, B, C due for white and D, E due for black. This method permits pairing A with C and B with D, which has 3 players with their due color. However, pairing A with D and B with E results in 4 players with their due color, which is higher than 3. So for an odd-cardinality score group, this method does not maximize the number of pairings where both players receive their due color.

4 Even-cardinality score groups

Assume we have an even size score group of n players. Let n_r be the number of players receiving their due. Then $n_r = n_r^w + n_r^w$ where n_r^w and n_r^b are the number of players receiving their due with white and black respectively.

Without loss of generality, since we know that either $n_r^w > \frac{n}{2}$ or $n_r^b > \frac{n}{2}$, we can assume that $n_r^w > \frac{n}{2}$.

Let P_1, ..., P_n be the players. After this method we will have pairings of P_1 with $P_{n/2+1}$, ..., $P_{n/2}$ with P_n.

Proposition 2. *Lower-bound on* n_r *is* $\frac{n}{2} + n_r^b$.

Proof. Assuming we satisfied the proposition, then since each pairing has at least one player due white, we know the player receiving white is due white. Without loss of generality, we can assume that P_1, ..., $P_{n/2}$ are due white and receive white, and each remaining player receives black. The second half, $P_{n/2+1}$, ..., P_n are due white, black, or neither, and receive black.

Since no player who is due black receives white, we know that each player due black must receive black. So we have n_r^b players due black and receiving black, and $\frac{n}{2}$ players due white receiving white. This means we have $\frac{n}{2} + n_r^b$ players receiving their due color. □

We've established a lower bound and we can show that it is the upper bound, to establish the process as optimizing the number of players receiving their due.

Proposition 3. *Upper-bound on players receiving their due is* $\frac{n}{2} + n_r^b$.

Proof. Now assume by any pairing method, at least $\frac{n}{2} + n_r^b + 1$ players receive their due color.

First assume that we have more than $\frac{n}{2}$ players receiving their due as white. This would mean that we have more than $\frac{n}{2}$ players playing white, which is a contradiction with the number of boards available. So we know that at most $\frac{n}{2}$ get their due as white.

The remaining $k > n_r^b$ players must receive their due as black, since $w \leq \frac{n}{2}$ received their due as white. But this means that since $k > n_r^b$, we have at least $n_r^b + 1$ players receiving black and due black. Earlier we defined n_r^b as the number of players which are due black, so this is a contradiction.

So we know that there may not be more than $\frac{n}{2} + n_r^b$ players who receive their due color. □

Thus since this forms both the lower and upper bound, we know that it maximizes the number of players which receive their due color.

5 Conclusion

As shown here, the Look Ahead method for pairing does achieve the optimal result for score groups which contain an even number of players. This is going to be practically true most of the time, but is an important consideration for tournament directors to keep in mind.